



1. Make sense of problems and persevere in solving them.

Mathematically proficient students:

- ❖ explain to themselves the meaning of a problem and look for entry points to its solution.
- ❖ analyze givens, constraints, relationships, and goals.
- ❖ make conjectures about the form and meaning of the solution attempt.
- ❖ consider analogous problems, and try special cases and simpler forms of the original problem.
- ❖ monitor and evaluate their progress and change course if necessary.
- ❖ transform algebraic expressions or change the viewing window on their graphing calculator to get information.
- ❖ explain correspondences between equations, verbal descriptions, tables, and graphs.
- ❖ draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- ❖ use concrete objects or pictures to help conceptualize and solve a problem.
- ❖ check their answers to problems using a different method.
- ❖ ask themselves, “Does this make sense?”
- ❖ understand the approaches of others to solving complex problems.

2. Reason abstractly and quantitatively.

Mathematically proficient students:

- ❖ make sense of quantities and their relationships in problem situations.
 - *decontextualize* (abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and
 - *contextualize* (pause as needed during the manipulation process in order to probe into the referents for the symbols involved).
- ❖ use quantitative reasoning that entails creating a coherent representation of quantities, not just how to compute them
- ❖ know and flexibly use different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- ❖ understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- ❖ make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- ❖ analyze situations by breaking them into cases
- ❖ recognize and use counterexamples.
- ❖ justify their conclusions, communicate them to others, and respond to the arguments of others.
- ❖ reason inductively about data, making plausible arguments that take into account the context
- ❖ compare the effectiveness of plausible arguments
- ❖ distinguish correct logic or reasoning from that which is flawed
 - elementary students construct arguments using objects, drawings, diagrams, and actions..
 - later students learn to determine domains to which an argument applies.
- ❖ listen or read the arguments of others, decide whether they make sense, and ask useful questions

4. Model with mathematics.

Mathematically proficient students:

- ❖ apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
 - In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
 - By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- ❖ simplify a complicated situation, realizing that these may need revision later.
- ❖ identify important quantities in a practical situation
- ❖ map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- ❖ analyze those relationships mathematically to draw conclusions.
- ❖ interpret their mathematical results in the context of the situation.
- ❖ reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. Use appropriate tools strategically.

Mathematically proficient students

- consider available tools when solving a mathematical problem.
- are familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools
- detect possible errors by using estimations and other mathematical knowledge.
- know that technology can enable them to visualize the results of varying assumptions, and explore consequences.
- identify relevant mathematical resources and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students:

- ❖ try to communicate precisely to others.
- ❖ use clear definitions in discussion with others and in their own reasoning.
- ❖ state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- ❖ specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- ❖ calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the context.
- ❖ In the elementary grades, students give carefully formulated explanations to each other.
- ❖ In high school, students have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students:

- ❖ look closely to discern a pattern or structure.
 - Young students might notice that three and seven more is the same amount as seven and three more.
 - Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for the distributive property.
 - In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$.
- ❖ step back for an overview and can shift perspective.
- ❖ see complicated things, such as some algebraic expressions, as single objects or composed of several objects.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- ❖ notice if calculations are repeated
- ❖ look both for general methods and for shortcuts.
- ❖ maintain oversight of the process, while attending to the details.
- ❖ continually evaluate the reasonableness of intermediate results.